Intermediate Microeconomics

Chapter 8: Slutsky Equation

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Changes in a Good's Price

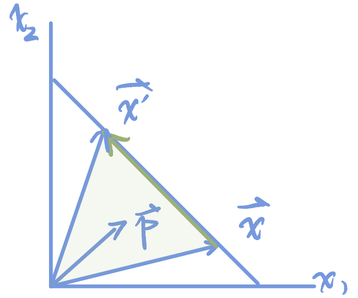
$p_1x_1 + p_2x_2 = m$

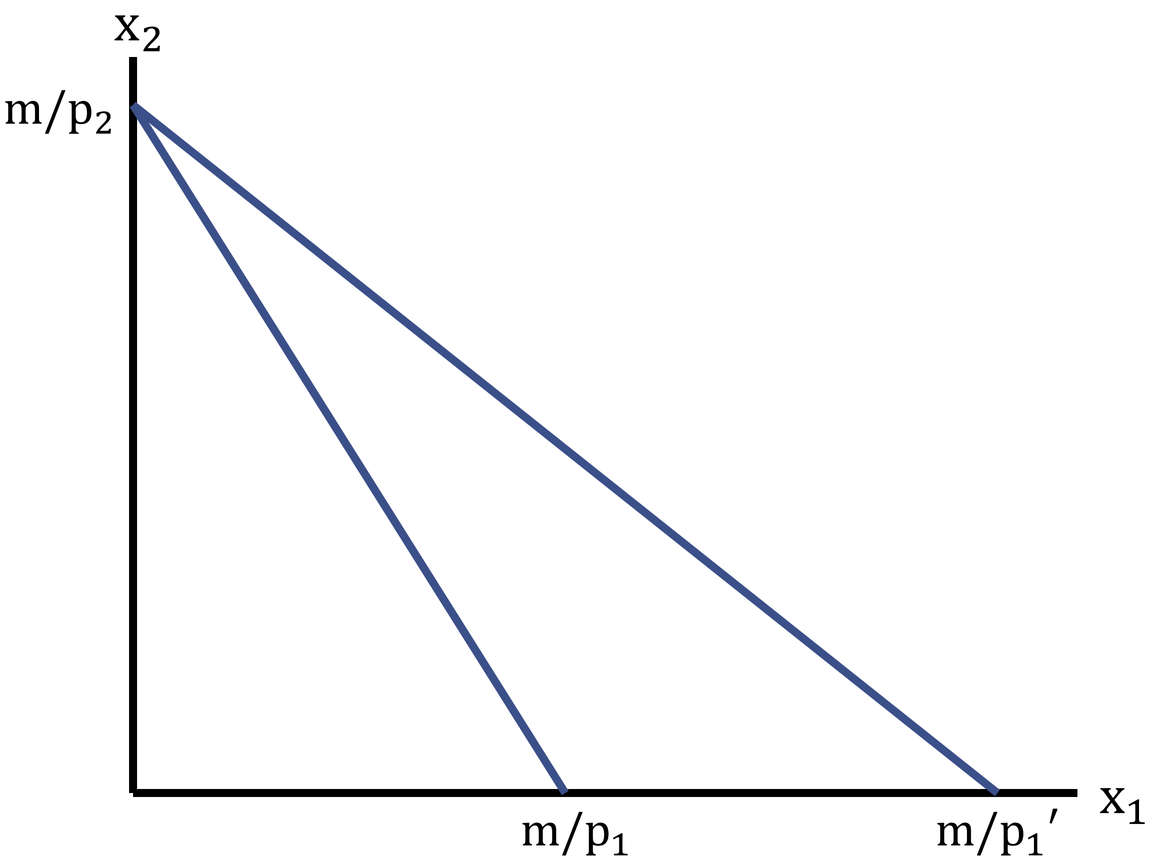
$\vec{p}\cdot\vec{x} = m$

$\vec{p}\cdot\vec{x'} = \vec{p}\cdot\vec{x}$

$\vec{p}(\vec{x'}-\vec{x}) = 0$

$\Rightarrow \vec{p} \perp (\vec{x'}-\vec{x})$, 已知 $\vec{p}$ $\Rightarrow$ slope.





Consumer's budget is \$m.

Lower price for good 1 pivots the budget line outwards.

$B(p_1, p_2, m) \to B(p_1', p_2, m - \Delta S) \to B(p_1', p_2, m)$

$\Delta S = m' - m$

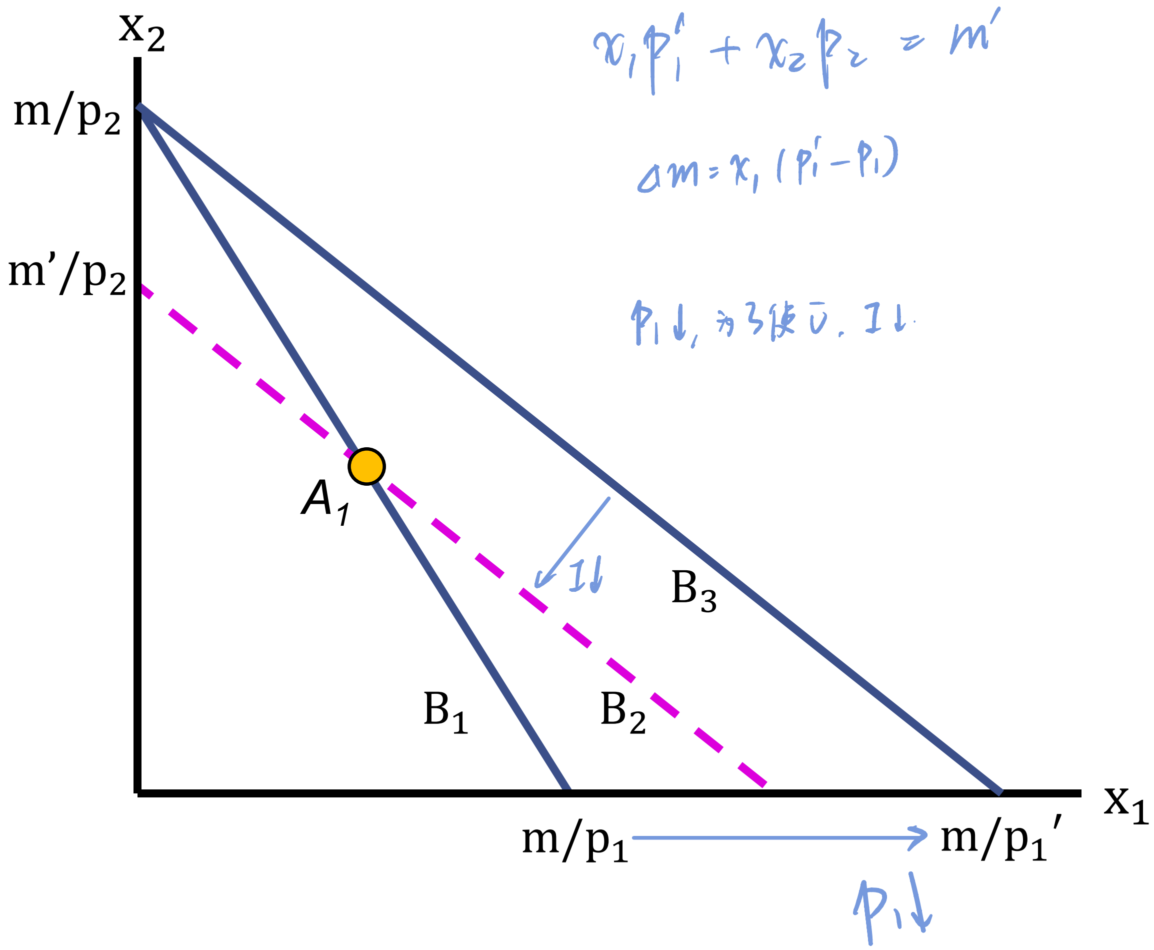
Changes in a Good's Price

$x_1 p_1 + x_2 p_2 = m$

$x_1 p_1' + x_2 p_2 = m'$

$\Delta m = x_1 (p_1' - p_1)$

$p_1 \downarrow$, 为了使 $\bar{U}$, $I \downarrow$.

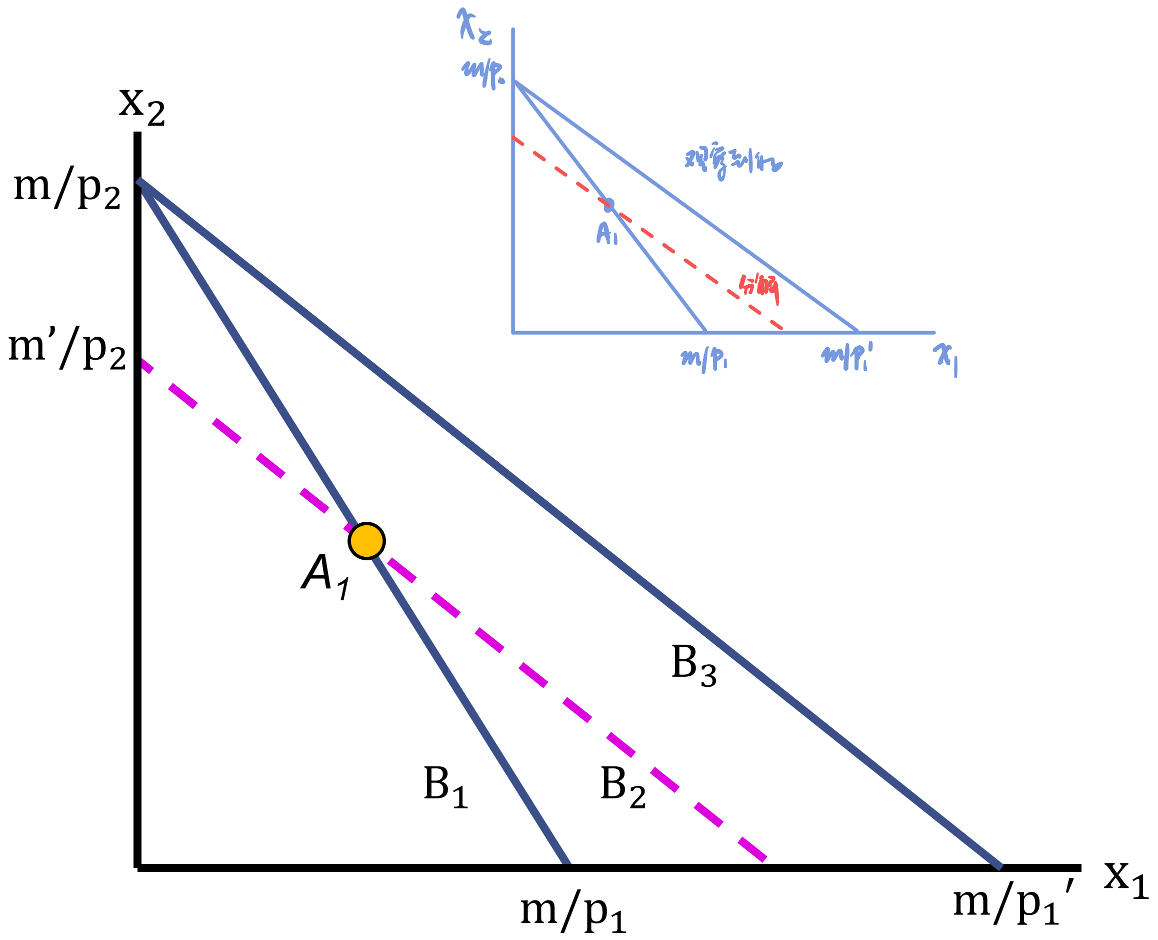


Consumer's budget is \$m.

Lower price for good 1 pivots the budget line outwards.

Only \$m' are needed to buy the original bundle at the new prices, as if the consumer's "real" income has increased by ___ ?

Changes in a Good's Price



Original budget: B_1 (p_1, p_2, m)

旋转：替代效应 ↓Substitution effect ↓

Pivoted budget: $B_2(p'_1, p_2, m-\Delta_s)$

平移：收入效应 ↓ Income effect ↓

Final (shifted) budget: $B_3(p'_1, p_2, m)$

Price effect

= Substitution effect + Income effect

Slutsky Equation

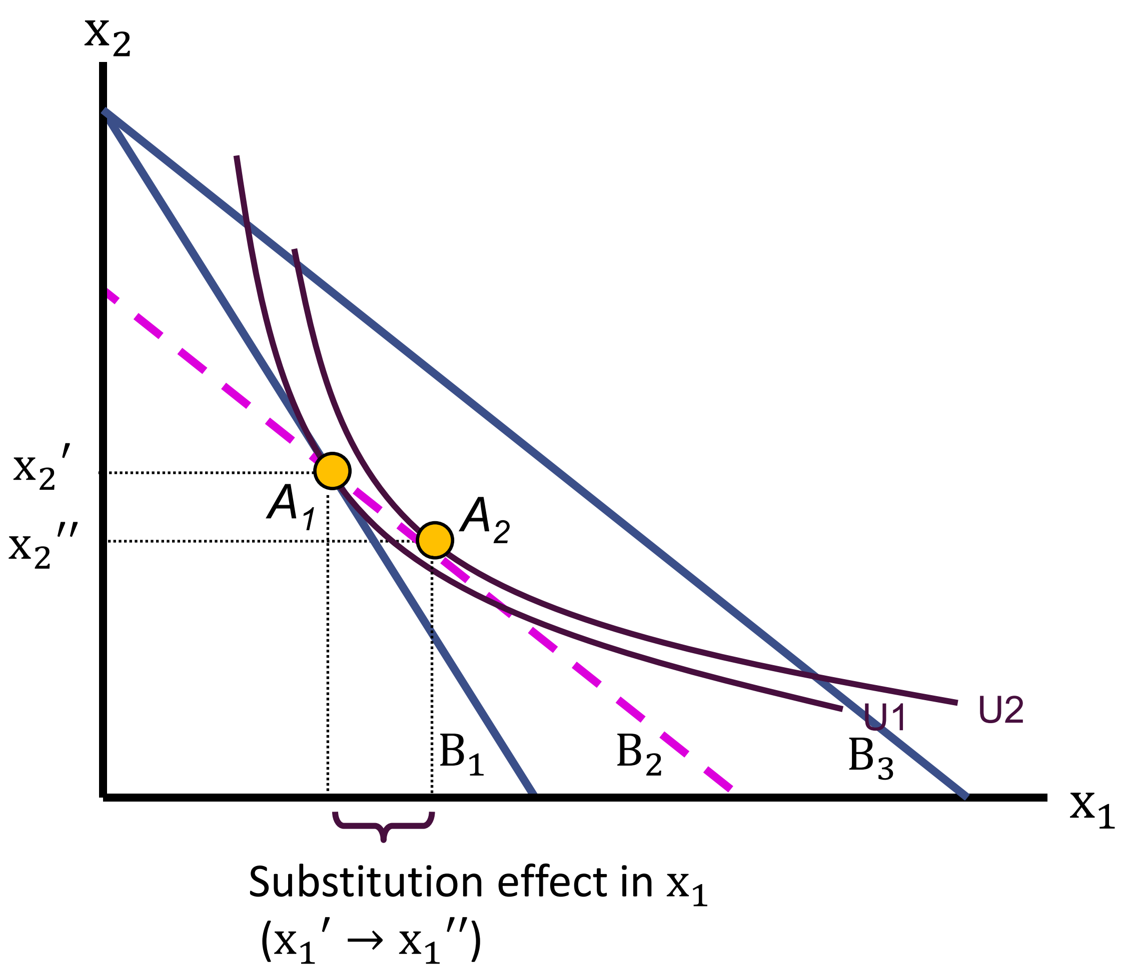
Price Change: "Pure" price change + income change

Price Effect (PE) Decomposition:

- Step 1: Change the consumption bundle through the Slutsky compensation method. This isolates the "pure" price change effect, referred to as the Substitution Effect (SE).
- Step 2: Remove the Slutsky compensation. This change represents the Income Effect (IE).

Slutsky Equation: Price Effect (PE) = Substitution Effect (SE) + Income Effect (IE)

Decompose Price Effect : Substitution Effect

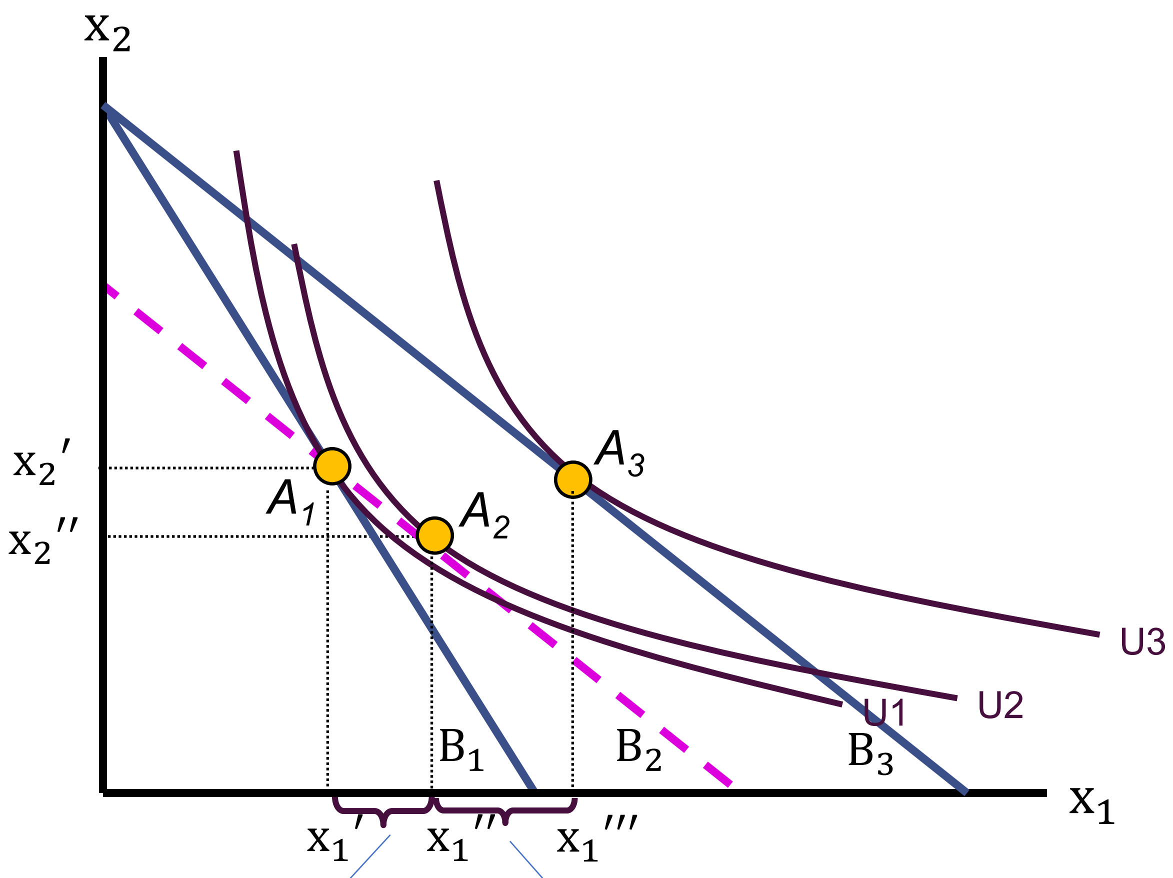


Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1.

$$A_1(x_1', x_2') \rightarrow A_2(x_1'', x_2'')$$

is the pure substitution effect (sometimes called the change in compensated demand).

Decompose Price Effect: Income Effect



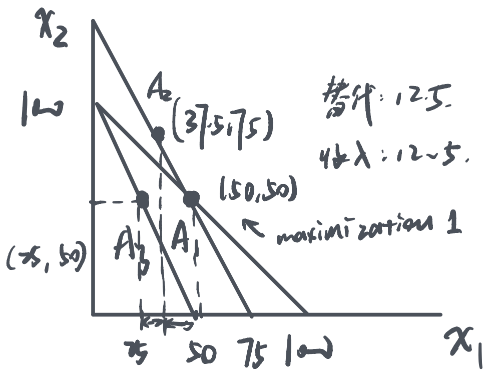
Substitution effect ($x_1' \rightarrow x_1''$)

Income effect ($x_1'' \rightarrow x_1'''$)

$A_2(x_1'', x_2'') \rightarrow A_3(x_1''', x_2''')$

is the income effect.

An Exercise



$x_1 = 50,\ x_2 = 50.$

$m = 100, p_1, p_2 = 1, u(x_1, x_2) = \sqrt{x_1 x_2}$

$2x_1 + x_2 = 100 \quad x_2 = 100 - 2x_1 \quad \sqrt{(100-2x_1)\,x_1} \quad x_1 = 25 \quad x_2 = 50$

Considering a change in good 1 price from 1 to 2 ($p'_1 = 2$), 0, 50

$\boxed{\Delta S = 50.}$

$2x_1 + x_2 = 150 \quad 15 \quad 75 \quad 0 \quad 2\overline{)75}\ 37.5 \quad 6 \quad 15$

(1) Δs (slutsky compensation)?

(2) Substitution effect | Income effect, on the demand for good 1?

1° 求 A_1

$\max \sqrt{x_1 x_2}$

s.t. $x_1 + x_2 = 100$

$\Rightarrow \begin{cases} x_1^* = \frac{\gamma m}{p_1} = 50 \\ x_2^* = \frac{(1-\gamma) m}{p_2} = 50 \end{cases}$

$p_1' x^* + p_2 x^* = m + \Delta s$

$\Rightarrow \Delta s = 50$

2° 求 A_2 (SE)

$\max \sqrt{x_1 x_2}$

s.t. $2x_1 + x_2 = 150$

$\begin{cases} x_1'^* = \cdots = 37.5 \\ x_2'^* = \cdots = 75 \end{cases}$

$\Delta x_1^s = x_1'^* - x_1^* = -12.5$

3° 求 A_3 (IE)

$\max \sqrt{x_1 x_2}$

s.t. $2x_1 + x_2 = 100$

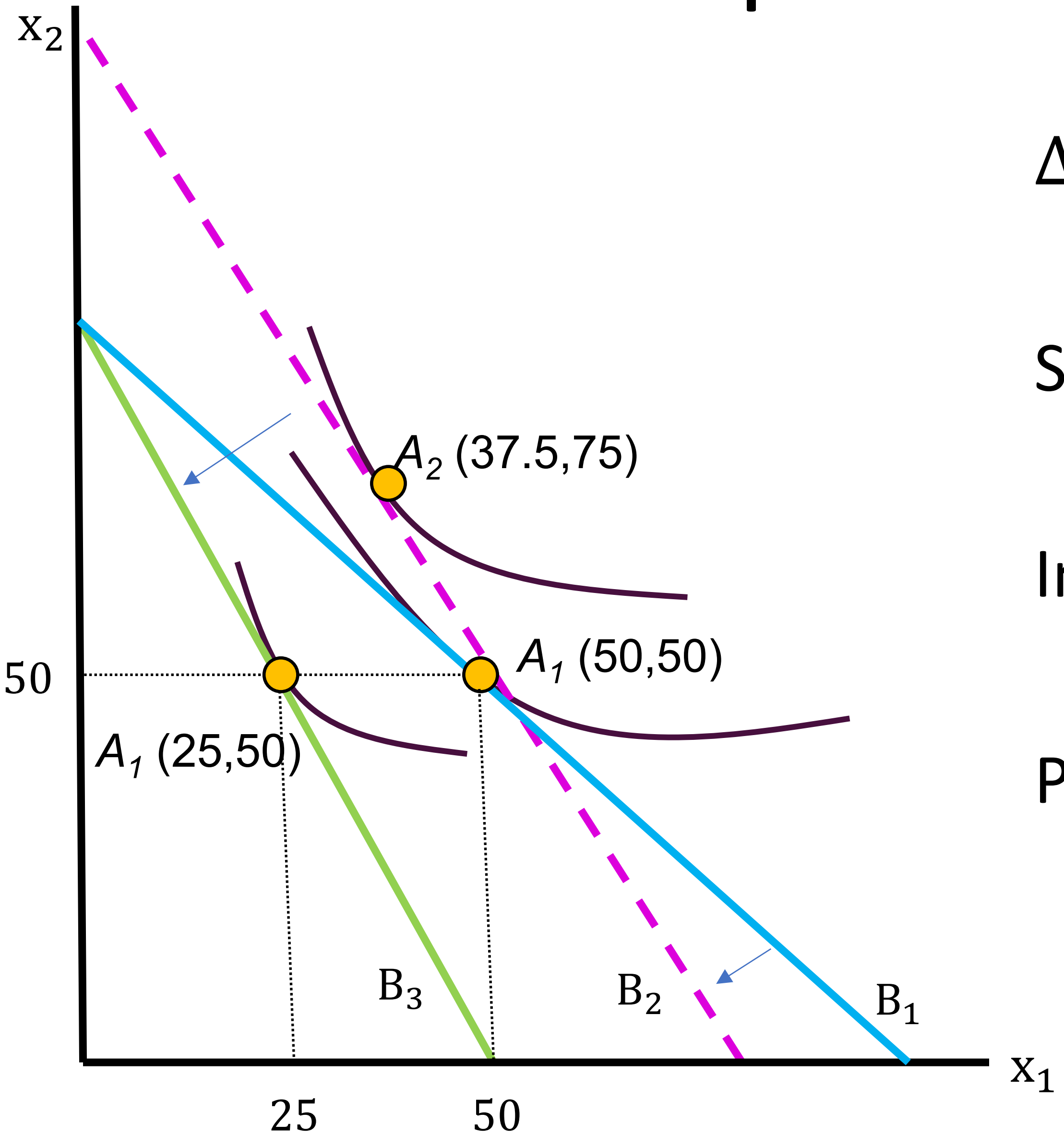
$\begin{cases} x_1''^* = \cdots = 25 \\ x_2''^* = \cdots = 50 \end{cases}$

$\Delta x_1^n = x_1''^* - x_1^* = -12.5$

$\Delta x_1 = \Delta x_1^s + \Delta x_1^n = -25$

12.5. 12.5.

An Exercise: Graphical Illustration



Δs=50[redacted]

Substitution effect Δx_1^s=-12.5

Income effect Δx_1^n=-12.5

Price effect= Δx_1^s+ Δx_1^n =-25

Revealed Preference* (Chapter 7)

Observing the choices of consumers can allow us to “recover” the preferences that lie behind those choices.

Weak Axiom of Revealed Preference (WARP)：

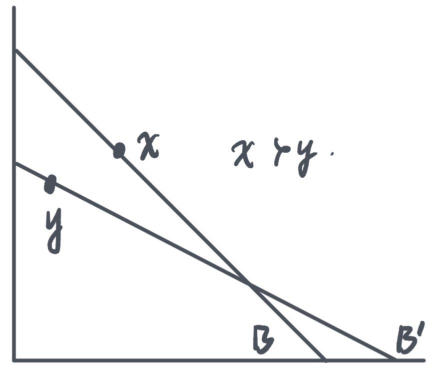
bundle

$$x, y \in B$$ 预算集

choose B

$$x \in C(B) \to x \succsim y$$

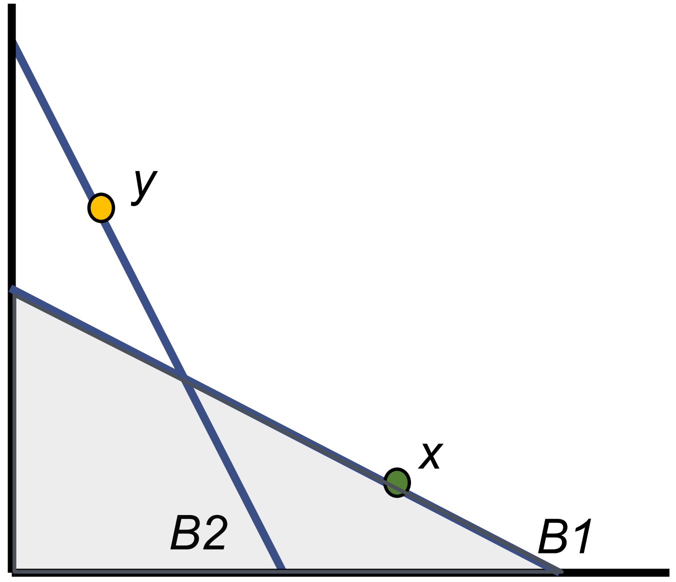
$$y \in C(B'), x \notin C(B') \to x \notin B'$$



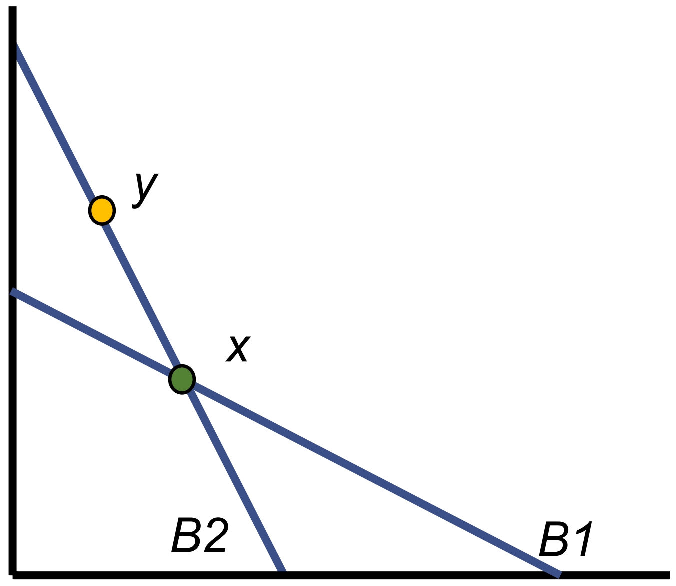
Revealed Preference

Does the consumer choice violate the Weak Axiom of Revealed Preference WARP?

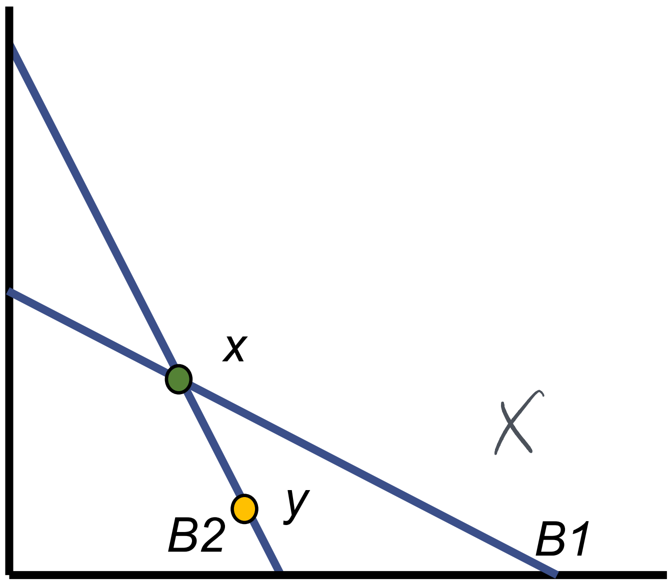
第一次选x，第二次选y．说明y不在x的预算集中．



Case 1

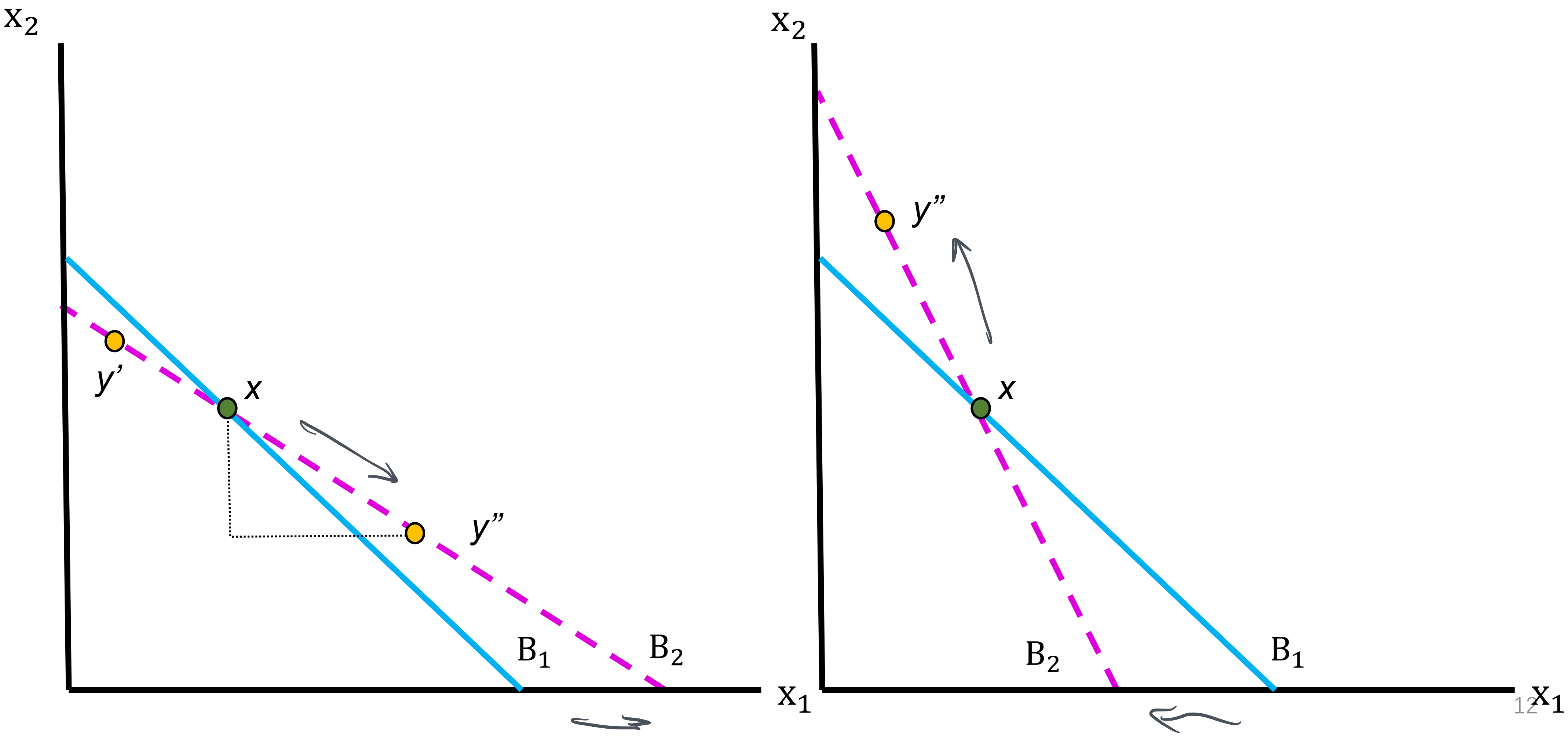


Case 2

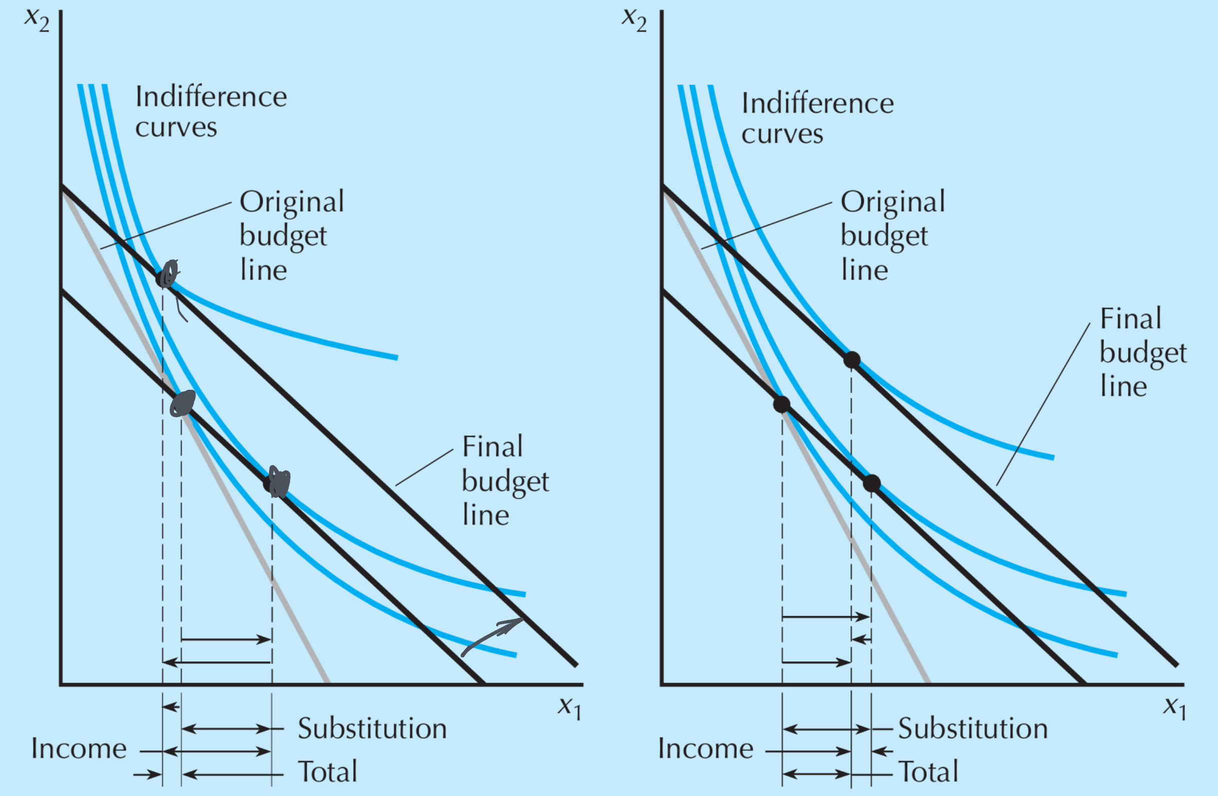


Case 3

Substitution Effect: Always Negative



Income Effect: Positive or Negative



A The Giffen case

B Non-Giffen inferior good

Inferior goods. Panel A shows a good that is inferior enough to cause the Giffen case. Panel B shows a good that is inferior, but the effect is not strong enough to create a Giffen good.

Slutsky Equation:	Price Effect=	Substitution Effect+	Income Effect
A Normal Good	(-)	(-)	(+)
An Inferior Good	(-)	(-)	(-)
A Giffen Good	(+)	(-)	(-)

P↓, I↑

Slutsky Equation

Slutsky Equation: Price Effect (PE) = Substitution Effect (SE) + Income Effect (IE)

PE SE − IE

$$\frac{\partial x_1(p_1, p_2, \bar{m})}{\partial p_1} = \frac{\partial x_1^s(p_1, p_2, \overline{x_1}, \overline{x_2})}{\partial p_1} - \frac{\partial x_1^m(p_1, p_2, \bar{m})}{\partial m}\overline{x_1}$$

Decompose Price Effect: Different Utility Functions (Perfect Substitutes)

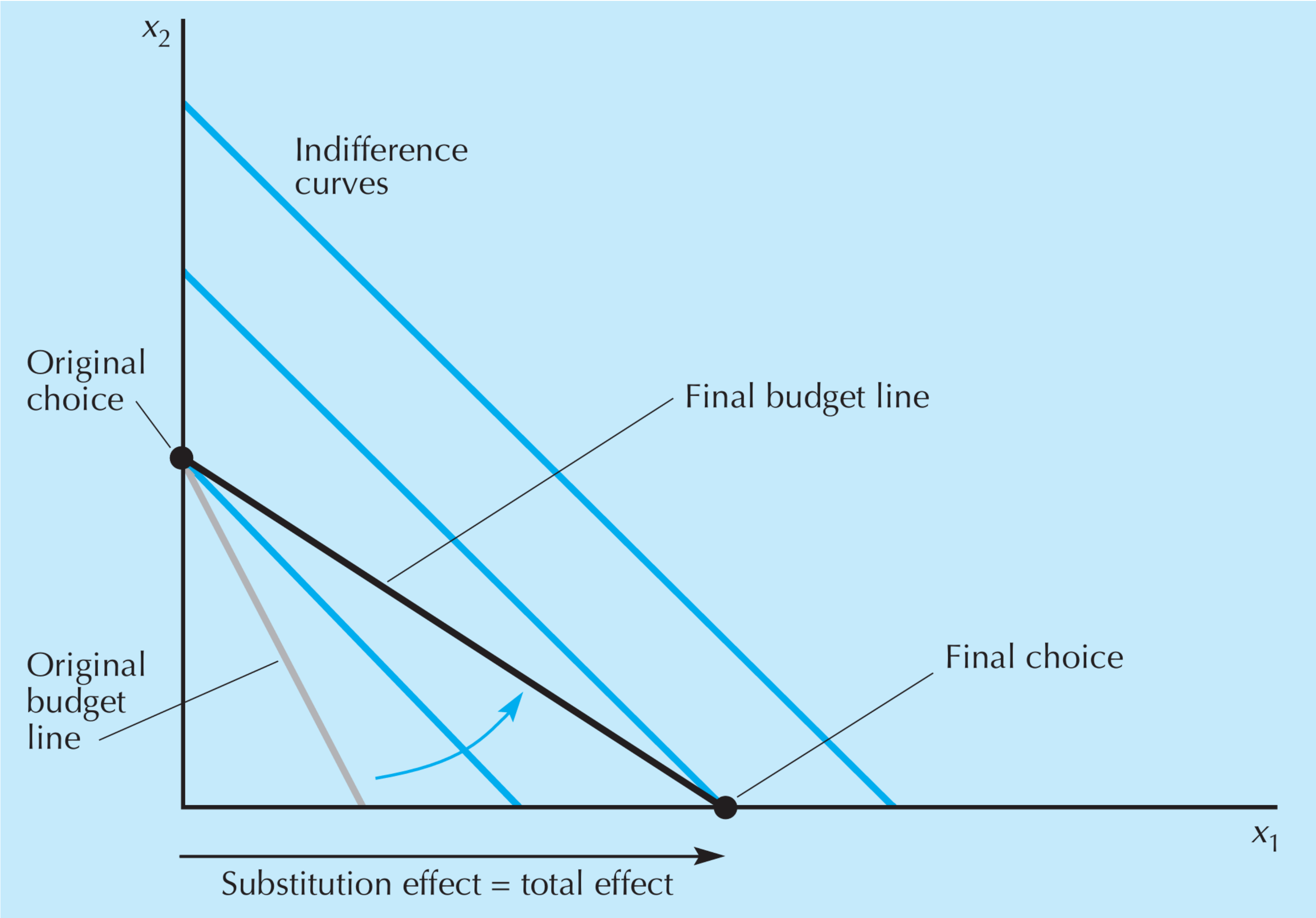
Take

$$u(x_1, x_2)=x_1 + x_2$$

Substitution effect=? ✓

Income effect=? 0

Decompose Price Effect: Different Utility Functions (Perfect Substitutes)



Decompose Price Effect: Different Utility Functions (Perfect Complements)

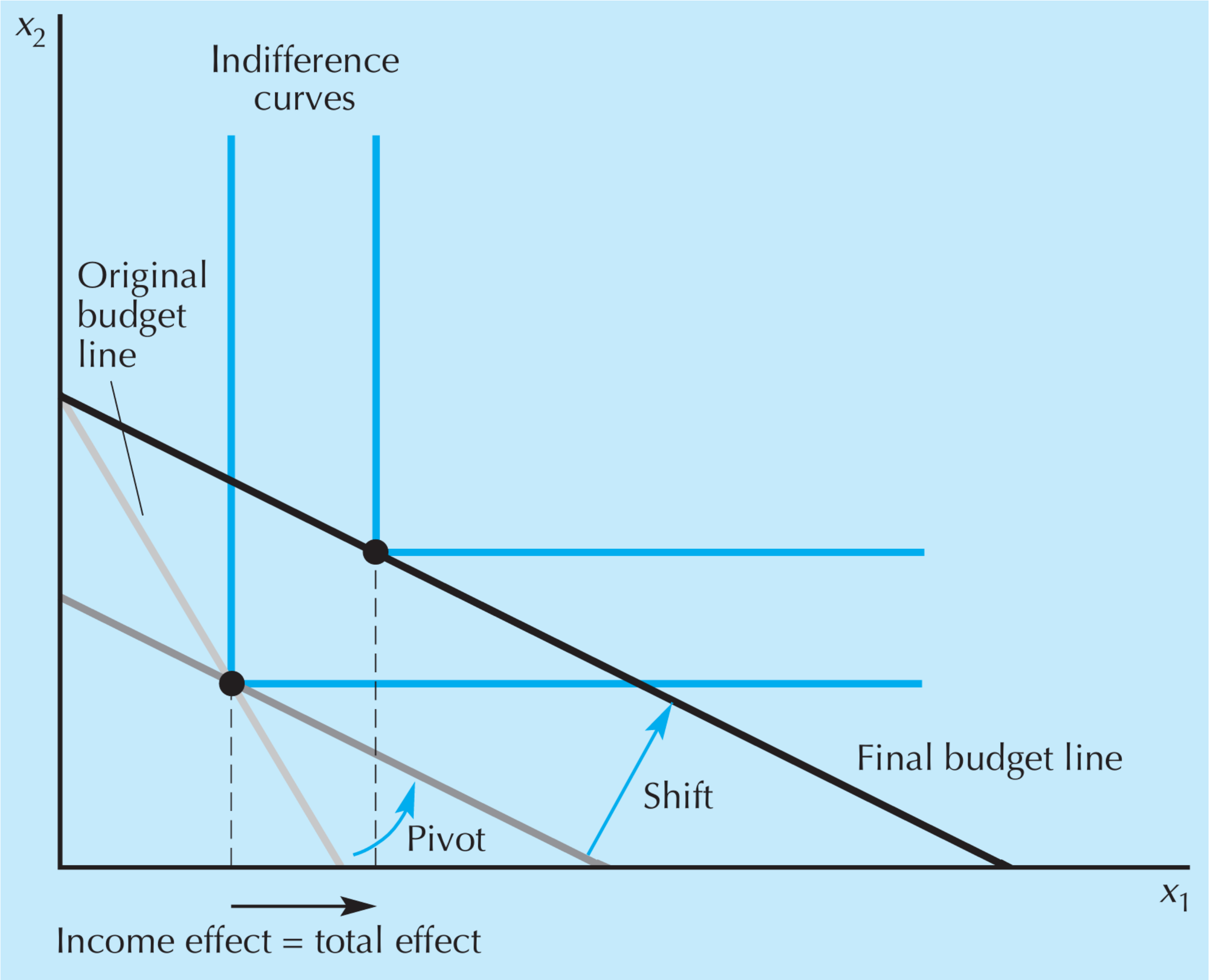
Take

$$u(x_1, x_2)=\min(x_1, x_2)$$

Substitution effect=? 0

Income effect=? ✓

Decompose Price Effect: Different Utility Functions (Perfect Complements)



Decompose Price Effect: Different Utility Functions (Quasilinear Preferences)

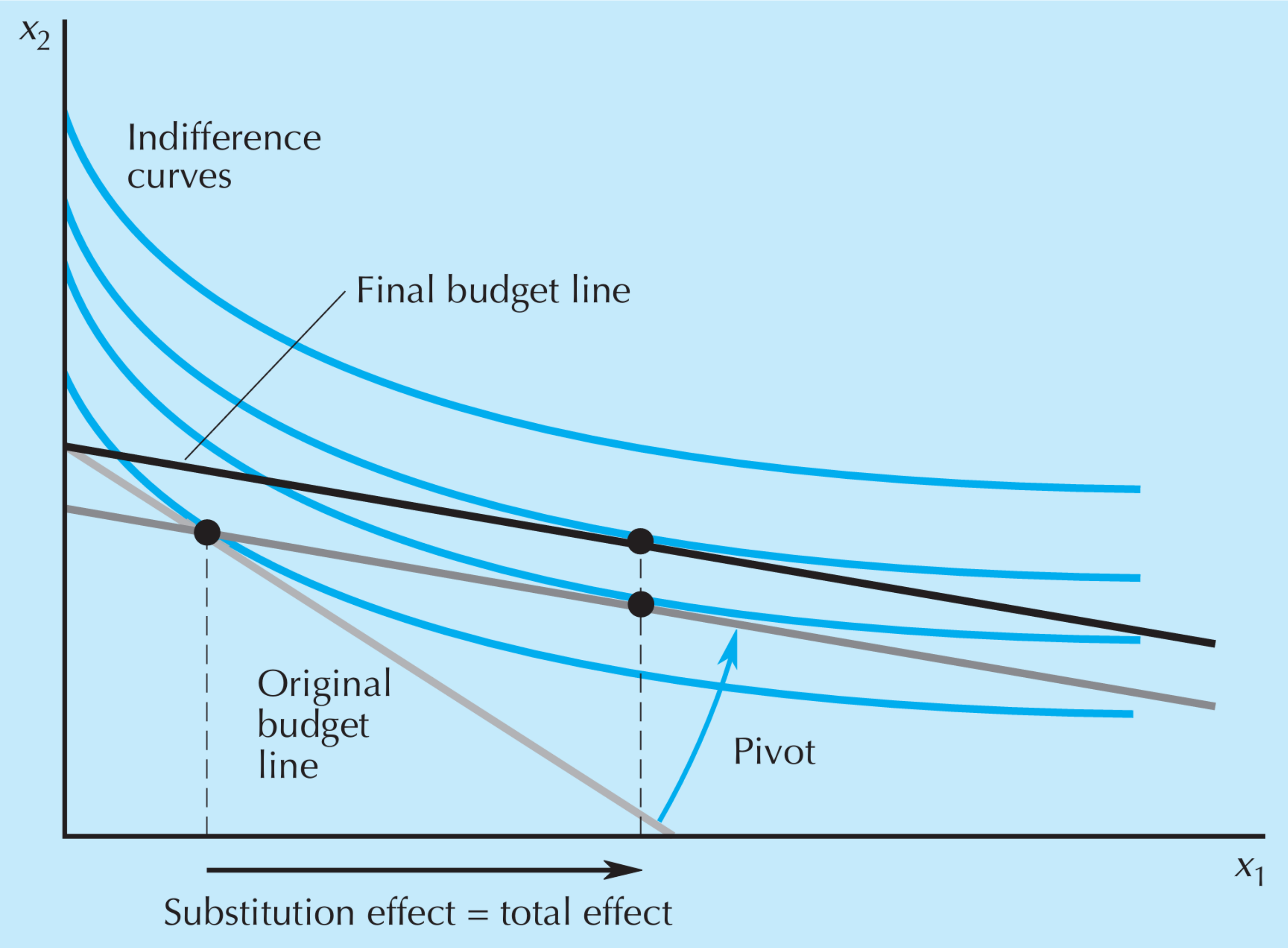
Take

$$u(x_1, x_2)=\sqrt{x_1} + x_2$$
$$p_1=1, p_2=1, m=10$$
$$p'_1=2$$

Substitution effect=?

Income effect=? 0

Decompose Price Effect: Different Utility Functions (Quasilinear Preferences)



Another Substitution Effect

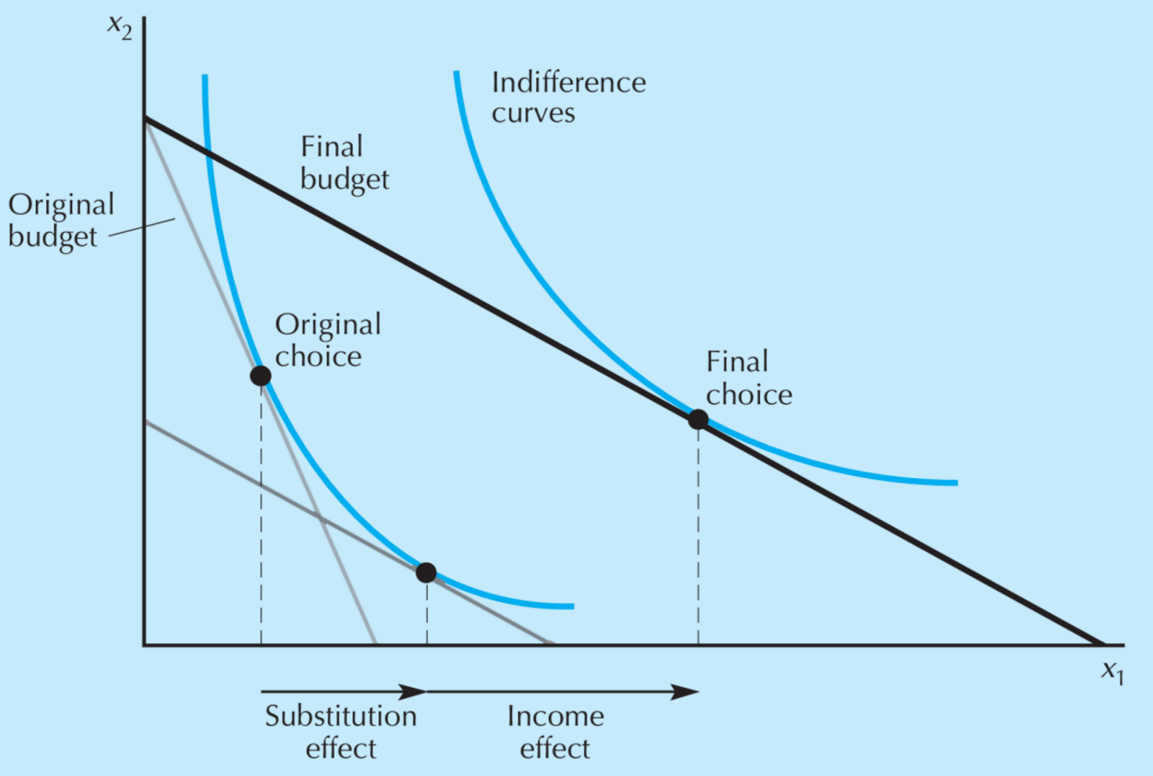
How the quantity demanded changes as a price changes in three different contexts:

1. Standard case: Holding income fixed

购买力固定，效用会上升。

2. Slutsky substitution effect: Holding purchasing power fixed
3. Hicks substitution effect: Holding utility fixed

Another Substitution Effect



The Hicks substitution effect. Here we pivot the budget line around the indifference curve rather than around the original choice.

An Exercise

$\mathrm{m} = 100, \mathrm{p}_{1,} \mathrm{p}_2 = 1, \mathrm{u}(\mathrm{x}_1, \mathrm{x}_2) = \sqrt{\mathrm{x}_1 \mathrm{x}_2}$

Considering a change in good 1 price from 1 to 2 ($\mathrm{p'}_1 = 2$),

(1) Δs (Hicks compensation)?

(2) Substitution effect | Income effect, on the demand for good 1?

$u = \sqrt{x_1 x_2}$

$p_1 = p_2 = 1, m = 100$

$\begin{cases} x_1^* = 50 \\ x_2^* = 50 \end{cases}$ $\bar{u} = 50$

$B_1 \to B_2$ $\max \sqrt{x_1 x_2}$

$2x_1 + x_2 = m'$

$\begin{cases} x_1^* = \frac{m'}{4} \\ x_2^* = \frac{m'}{2} \end{cases}$

$u(m') = \frac{\sqrt{2}}{4} m' = \bar{u} = 50$

$m' = 100\sqrt{2}$

$\Delta h = 100(\sqrt{2} - 1)$

$\begin{cases} x_1^{*\prime} = 25\sqrt{2} \\ x_2^{*\prime} = 50\sqrt{2} \end{cases}$

$B_2 \to B_3$:

$\max \sqrt{x_1 x_2}$

$2x_1 + x_2 = 100$

$\begin{cases} x_1^{*\prime\prime} = \frac{100}{3} \\ x_2^{*\prime\prime} = \frac{200}{3} \end{cases}$

Summary

When the price of a good decreases, there will be two effects on consumption. The change in relative prices makes the consumer want to consume more of the cheaper good. The increase in purchasing power due to the lower price may increase or decrease consumption, depending on whether the good is a normal good or an inferior good.

The change in demand due to the change in relative prices is called the substitution effect; the change due to the change in purchasing power is called the income effect.

The substitution effect is how demand changes when prices change and purchasing power is held constant, in the sense that the original bundle remains affordable. To hold real purchasing power constant, money income will have to change. The necessary change in money income is given by $\Delta m = x_1 \Delta p_1$

The Slutsky equation says that the total change in demand is the sum of the substitution effect and the income effect